

Решение задачи о СДНФ булевой функции

Задача. Используя СКНФ, найдите наиболее простую формулу алгебры высказываний от четырех переменных, принимающую значение 0 на следующих наборах значений переменных, и только на них:

$$F(1,1,1,0) = F(1,1,0,1) = F(1,0,1,1) = F(0,1,1,1) = F(1,0,0,1) = 0.$$

Решение. Запишем СКНФ функции по данным задачи:

$$F = (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee \bar{x}_2 \vee x_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee \bar{x}_3 \vee \bar{x}_4)(x_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee x_3 \vee \bar{x}_4).$$

Упростим по возможности полученное выражение:

$$\begin{aligned} F &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee \bar{x}_2 \vee x_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee \bar{x}_3 \vee \bar{x}_4)(x_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee x_3 \vee \bar{x}_4) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee (\bar{x}_3 \vee x_4))(x_3 \vee \bar{x}_4)((\bar{x}_1 \vee \bar{x}_2)(\bar{x}_1 \vee x_2) \vee \bar{x}_3 \vee \bar{x}_4)(x_1 \vee \bar{x}_2 \vee (\bar{x}_3 \vee x_4))(x_3 \vee \bar{x}_4) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \bar{x}_4 \vee x_3 x_4)(\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee \bar{x}_4) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \bar{x}_4 \vee x_3 x_4)(\bar{x}_1 \bar{x}_2 (\bar{x}_1 \vee x_2 \vee \bar{x}_4) \vee x_1 x_2 (\bar{x}_1 \vee x_2 \vee \bar{x}_4) \vee \bar{x}_3 (\bar{x}_1 \vee x_2 \vee \bar{x}_4) \vee \bar{x}_4 (\bar{x}_1 \vee x_2 \vee \bar{x}_4)) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \bar{x}_4 \vee x_3 x_4) \cdot \\ &= \left((\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \bar{x}_2 \vee x_1 x_2 x_4) \vee (x_1 x_2 \bar{x}_1 \vee x_1 x_2 x_2 \vee x_1 x_2 x_4) \vee (\bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4 x_3) \vee \bar{x}_4 \right) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \bar{x}_4 \vee x_3 x_4) \cdot (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4 x_3 \vee \bar{x}_4) = \\ &= (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \bar{x}_4 \vee x_3 x_4) \cdot (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4) = \\ &= \left(\begin{aligned} & \left(\bar{x}_1 (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4) \vee \bar{x}_2 (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4) \vee \right. \\ & \left. \vee \bar{x}_3 \bar{x}_4 (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4) \vee x_3 x_4 (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_1 \bar{x}_3 \vee x_2 \bar{x}_3 \vee \bar{x}_4) \right) = \\ & \left(\begin{aligned} & (\bar{x}_1 \bar{x}_1 \bar{x}_2 \vee \bar{x}_1 \bar{x}_1 x_2 \vee \bar{x}_1 \bar{x}_1 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_4) \vee (\bar{x}_1 x_2 \bar{x}_2 \vee x_1 x_2 \bar{x}_2 \vee \bar{x}_1 x_2 \bar{x}_3 \vee x_2 x_2 \bar{x}_3 \vee x_2 x_4) \vee \right. \\ & \left. \vee (\bar{x}_1 x_2 x_3 x_4 \vee x_1 x_2 x_3 x_4 \vee \bar{x}_1 x_3 x_3 x_4 \vee x_2 x_3 x_3 x_4 \vee x_4 x_3 x_4) \vee \right. \\ & \left. \vee (\bar{x}_1 x_2 x_3 x_4 \vee x_1 x_2 x_3 x_4 \vee \bar{x}_1 x_3 x_3 x_4 \vee x_2 x_3 x_3 x_4 \vee x_4 x_3 x_4) \right) = \end{aligned} \right) = \\ &= \left((\bar{x}_1 \bar{x}_2 \vee \bar{x}_1 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_4) \vee (\bar{x}_1 \bar{x}_2 \vee x_1 x_2 \bar{x}_3 \vee x_2 x_4) \vee \bar{x}_3 \bar{x}_4 \vee (x_1 x_2 x_3 x_4 \vee x_1 x_2 x_3 x_4) \right) = \\ &= (\bar{x}_1 \bar{x}_2 \vee \bar{x}_1 \bar{x}_3 \vee \bar{x}_1 x_4 \vee x_2 x_4 \vee \bar{x}_3 \bar{x}_4 \vee x_1 x_2 x_3 x_4) = \bar{x}_1 \bar{x}_2 \vee \bar{x}_1 \bar{x}_3 \vee \bar{x}_1 x_4 \vee x_2 x_4 \vee \bar{x}_3 \bar{x}_4 \vee x_1 x_2 x_3 x_4. \end{aligned}$$

Получили $F = \bar{x}_1 \bar{x}_2 \vee \bar{x}_1 \bar{x}_3 \vee \bar{x}_1 x_4 \vee x_2 x_4 \vee \bar{x}_3 \bar{x}_4 \vee x_1 x_2 x_3 x_4$.